Active Cancellation of Magnetic Noise

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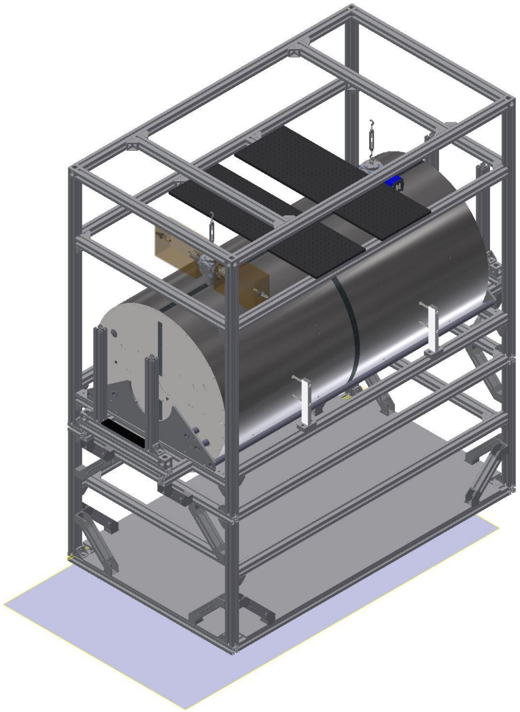


Imperial College London

Motivation

Task: Reduce Magnetic noises inside the Mu-metal shield

Challenges:

- Initial use of Independent Component Analysis (ICA) had suboptimal results.
- Machine learning for blind source separation is too complicated to achieve

Current Focus:

- Shift to dynamic compensation techniques.
- Offers a promising direction for reducing noise.

Dynamic Compensation

Overview

Noises are external at sub 1Hz

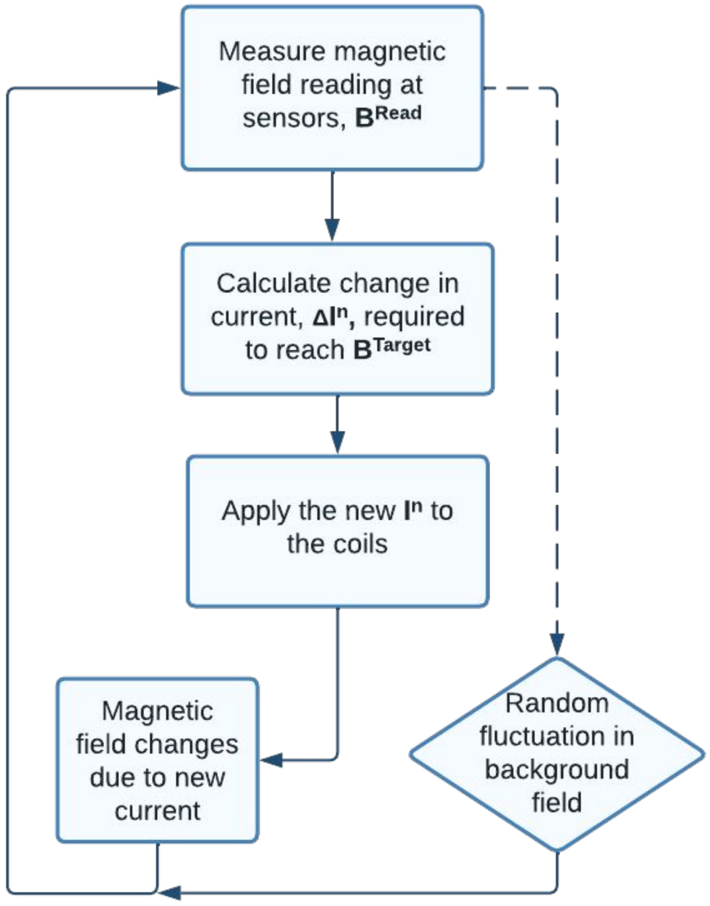
6 compensation coils, 2 along each axis

Up to 30 fluxgate magnetometers

PI feedback control loop

Compensating power increases with more detectors

Implemented with 1 coil and 1 detector



Dynamic Compensation

Proportionality factor matrix

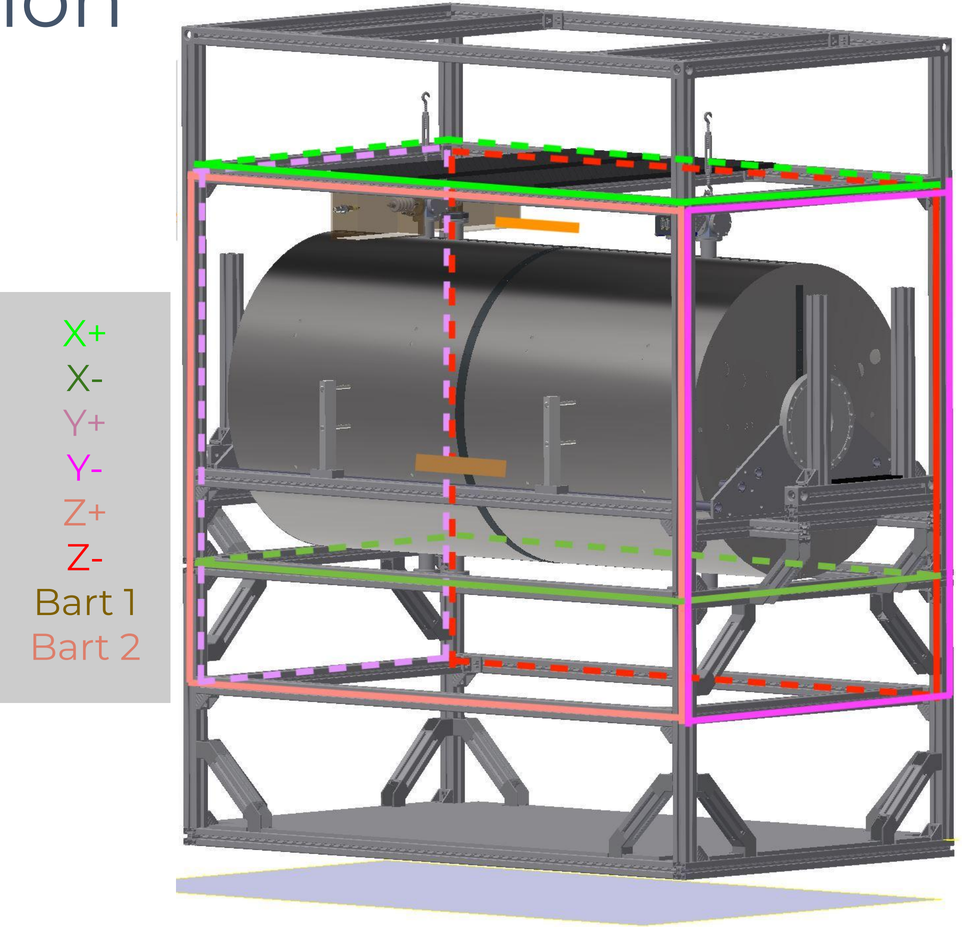
Each fluxgate sensor has a linear response to current changes in each of the six coils. Proportionally factor matrix defined as:

$$B_k = \sum_j M_{kj} I_j$$

Has unit of nT/A

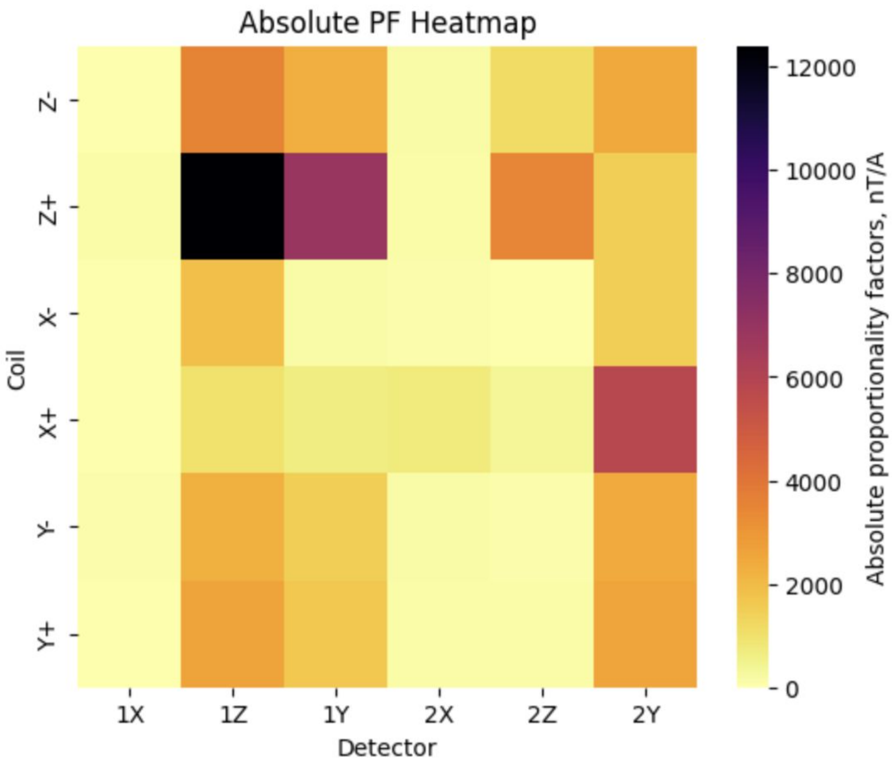
The compensating current is calculated by inverting *M*

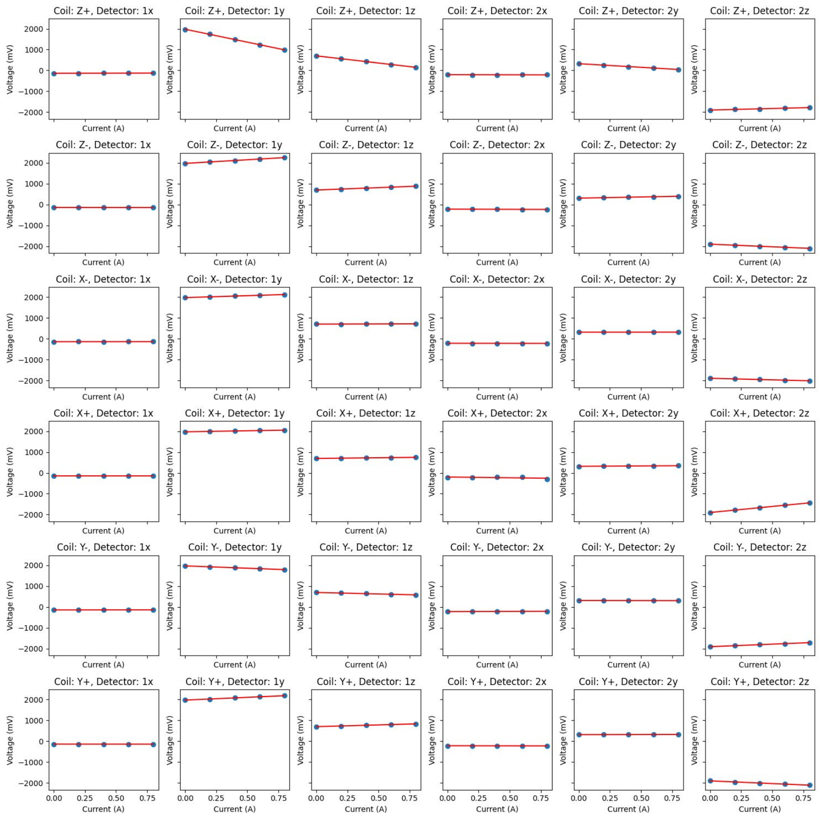
$$\Delta I_j = \sum_k M^{-1}_{jk} \cdot (B_k^{target} - B_k^{read})$$



Dynamic Compensation

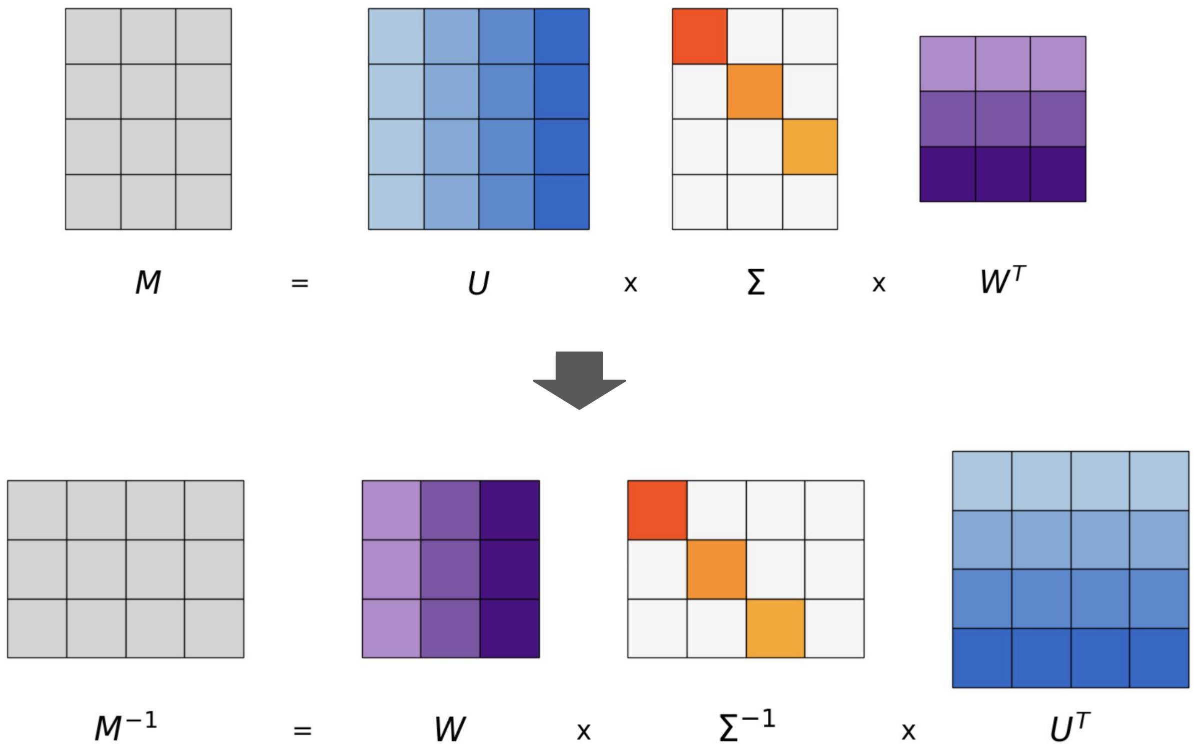
Proportionality factor matrix





Matrix Inversion: The Pseudoinverse

Done via Singular Value Decomposition



Matrix Inversion: The Pseudoinverse

Regularisation Parameter

- Our matrix M is ill-conditioned i.e has a high condition number
- This is a measure of the sensitivity in the output to changes and errors in the input
- Condition number is also given by the ratio of the largest to smallest singular value

$$\kappa(M) = \frac{\sigma_1}{\sigma_n}$$

- If there is a small σ_i, M will be almost singular and therefore difficult to invert
- We use Tikhonov regularisation to increase numerical stability

$$\Sigma_{ii}^{-1} = \frac{1}{\sigma_i} \longrightarrow \frac{\sigma_i}{\sigma_i^2 + \beta^2}$$

- Results in a less accurate pseudoinverse, but improves computational ease

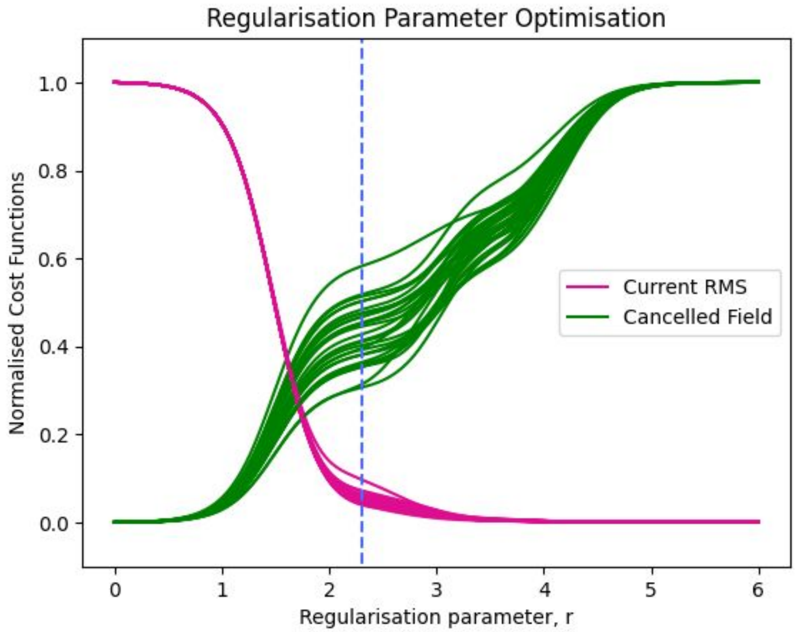
Matrix Inversion: the Pseudoinverse

Choosing the right Regularisation Parameter

A suitable regularisation parameter is required to create a compromise between finding an accurate pseudoinverse, and reducing the impact of errors on our calculations.

Created a simulation to compare these two factors and aim to minimise them both.

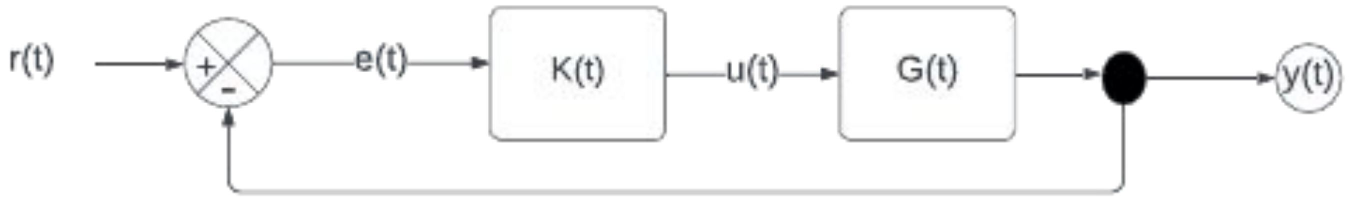
This gave an optimal r = 2.036, where $\beta = 10^r$. Ultimately, this needs to be verified experimentally.



Feedback Loops and Control Theory

Overview

- Design a feedback loop to actively cancel magnetic noise based on principles of control theory.
- A general feedback loop will include:
    - y(t)- Measured output value of this quantity
    - r(t) - Target for our controlled quantity
    - e(t) - Error given by r(t) - y(t)
    - K(t)- Control Law
    - u(t) - Response to control law
    - G(t)- System



Feedback Loops and Control Theory

PID control

To reduce the error, a PID control law is implemented:

$$u(t) = K_p \cdot e(t) + K_i \cdot \int_0^t e(\tau)\, d\tau + K_d \cdot \frac{de(t)}{dt}$$

P- Proportional Control, Corrects for current error

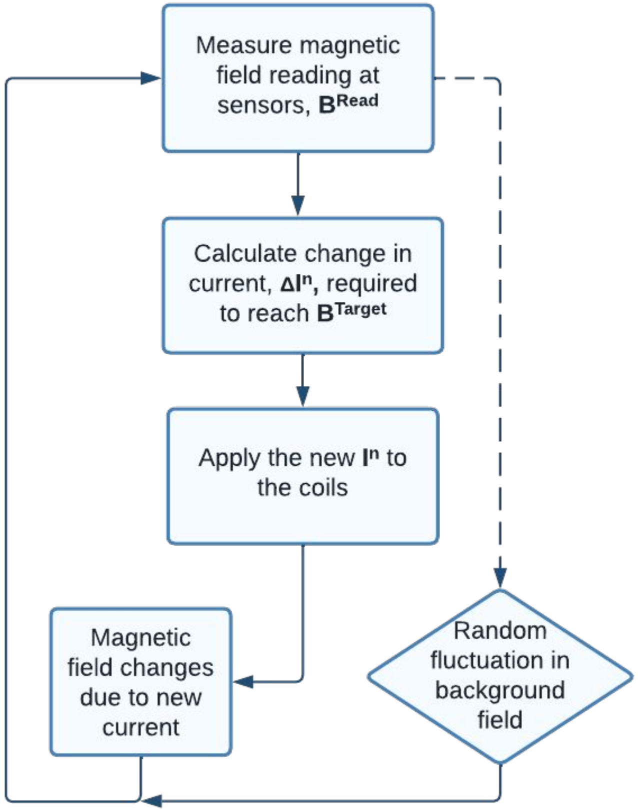
I- Integral Control, Uses cumulative value of the errors

D-Derivative Control, Predicts future trend of error

Experiment and Method

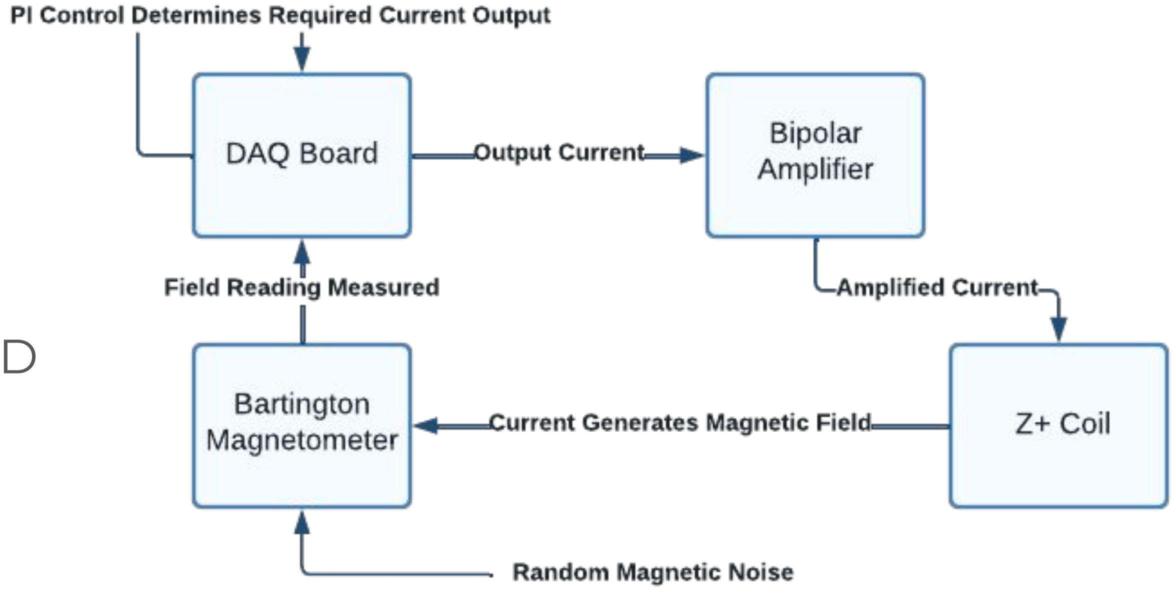
$$I_j^n = K_j^P \cdot \Delta I_j^n + K_j^I \cdot \sum_{t=0}^{n} \Delta I_j^t$$

- We update the current in each coil according to our PI control law
- Each coil has its own PI tuning parameters
- No derivative control - D increases noise



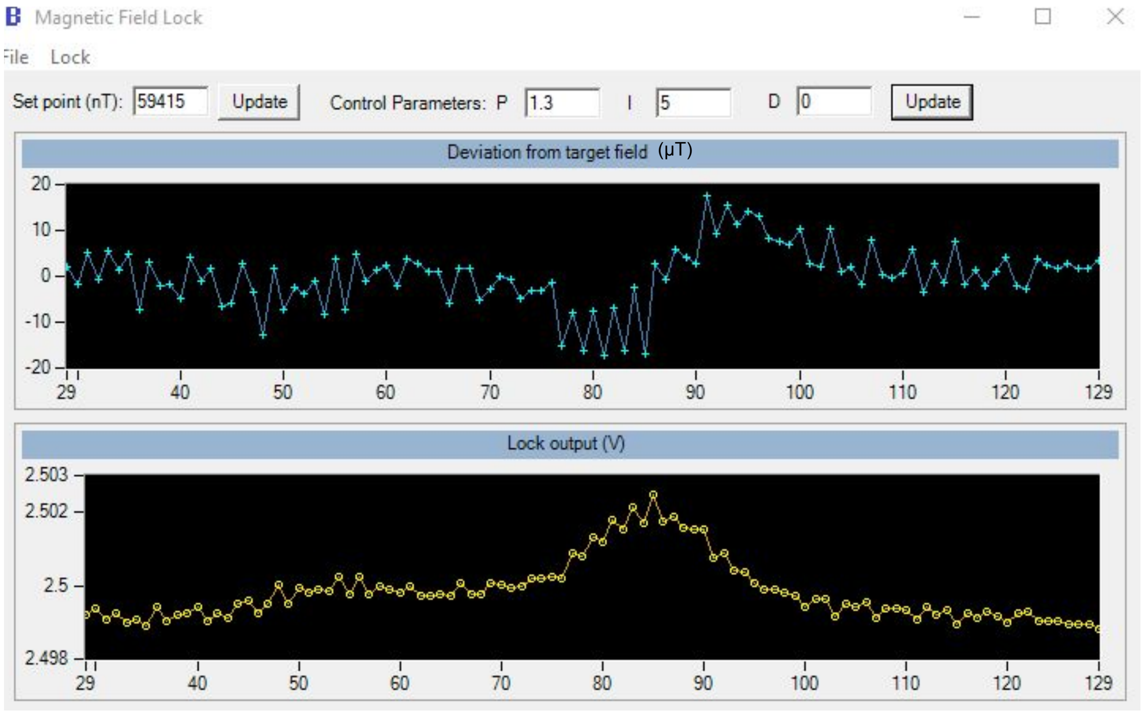
Experiment and Method

- Find optimal tuning parameters
- Choose appropriate SAMPLE_CLOCK_RATE and SAMPLE_MULTI_READ



Results
Magnetic Field Lock

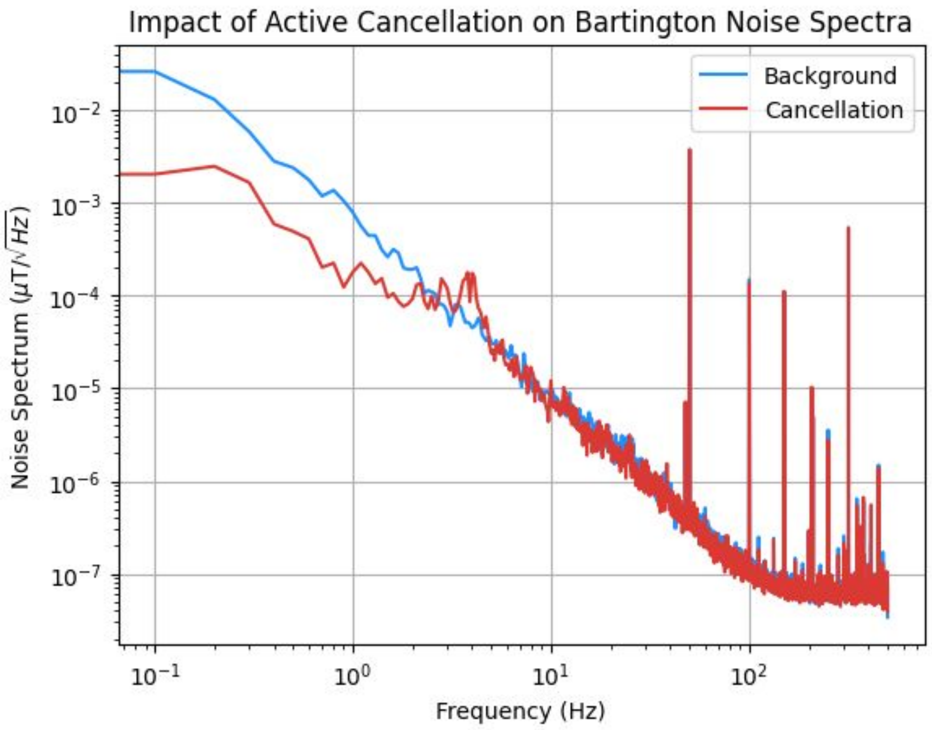
Visualisation of PI control for locking to a target magnetic field



Results
Bartington

- Clear reduction in noise below 1 Hz. At 0.1 Hz, the noise is reduced by a factor of 13 with cancellation

- We see a bump at around 4 Hz, this is the upper limit to which we can cancel.

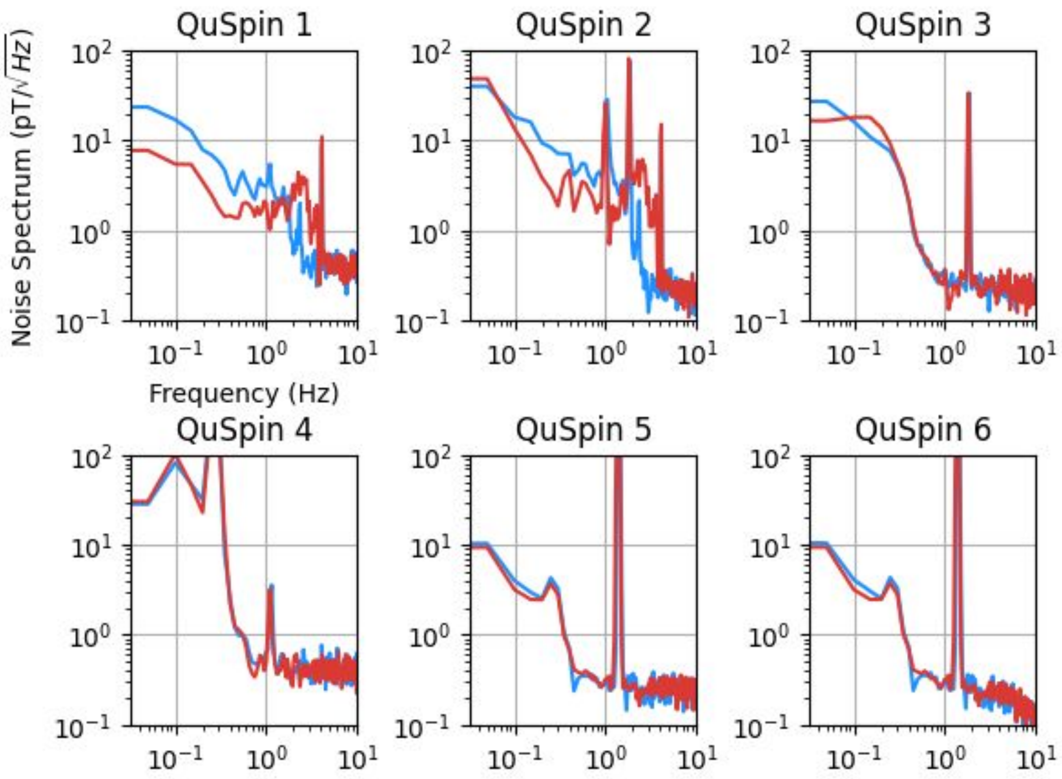


Results

QuSpins

- Looking now at internal noise
- Some QuSpins see improvements while others don't.
- Ratios at 0.1 Hz range from reductions of 0.80 to 3.1 with cancellation
- Little correlation between QuSpin position and effect of cancellation

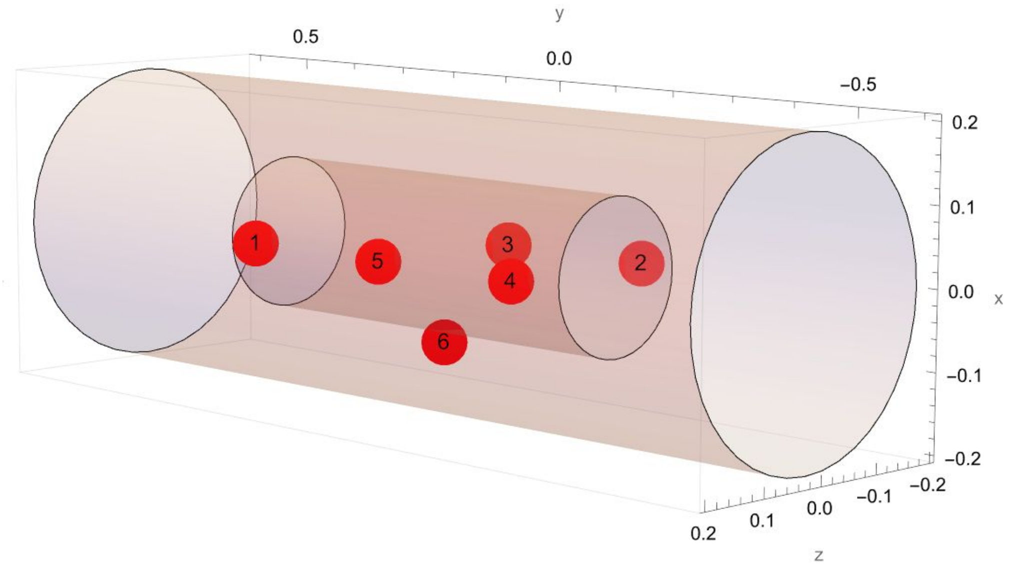
Impact of Active Cancellation on QuSpin Noise Spectra



Results

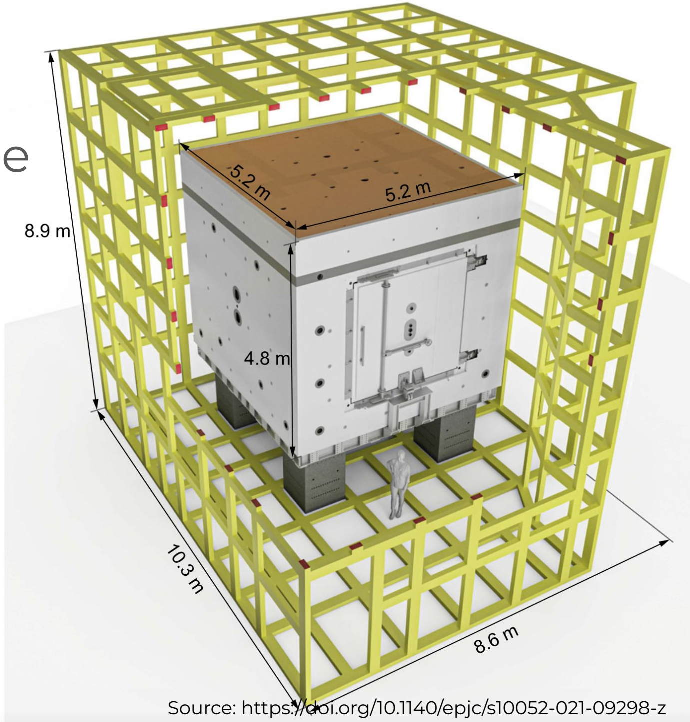
QuSpins

QuSpin	Cancellation Ratio at 0.1 Hz
1	3.1
2	1.3
3	0.89
4	0.80
5	1.26
6	1.29



Improvements & Next Steps

- Extend the setup to multiple detectors and coils
- Include a system that automatically finds the PF matrix
- Include gradient coils to produce specific gradients



Source: https://doi.org/10.1140/epjc/s10052-021-09298-z

Thank You

Regularisation Parameter simulation

- Generate a vector of K random magnetic field values, $\overrightarrow{B}^{rand}$
- Use M^{-1} to calculate $\overrightarrow{I}^{Sim}$
- We use Γ as a proxy cost function for the input variations

$$\Gamma(r) = \sqrt{\frac{1}{6}\sum_{j=1}^{6}(I_j^{Sim}(r))^2}$$

$$B_k^*(r) = B_k^{rand} + \sum_j M_{kj} \cdot I_j^{Sim}(r)$$

$$b^* = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(B_k^*)^2} \qquad b = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(B_k^{Rand})^2}$$

$$R = \frac{b^*}{b}$$

- R is a measure of how well the cancellation works, with R = 0 being perfect cancellation

Regularisation Parameter simulation

Γ and R were normalised to [0,1], and then we aimed to minimise them simultaneously.

This was achieved by summing R and Γ for each element in $\mathbf{B}^{\mathbf{Rand}}$, and then finding the r value corresponding to the minimum.

Varying the number of elements in $\mathbf{B}^{\mathbf{Rand}}$ had little impact on the optimal value of r