Measuring capacitance using an oscilloscope

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***Abstract*—The capacitance of an electrolytic and a ceramic capacitor are calculated using the discharge decay and phase shift methods. The calculated capacitance for the electrolytic capacitor is $1.03 \pm 0.01mF$ and that for the ceramic capacitor is $539 \pm 10pF$ For the phase shift method, a sine wave is used to drive the ceramic capacitor with the resistance and capacitance of the oscilloscope also being considered. The calculated capacitance for the same ceramic capacitance is $330 \pm 20pF$, showing that the capacitance calculated by the discharge decay method is higher than the more appropriate phase shift method. Finally, a low-pass filter model is suggested for calculating the capacitance, yielding a result of $440 \pm 20pF$.**

I. Introduction

Capacitors are able to store energy in an electric field. A capacitor gets charged when a charge Q from one electrical conductor is transferred to the other conductor, resulting one side to have a charge +Q and the other -Q. The capacitance, C, is defined to be Q/V where V is the potential difference between the electrical conductors. The first capacitor, the Leyden jars, was introduced in the 18th century by researchers who wanted to investigate the effect of charges of animals.[1] In this investigation, electrolytic and ceramic capacitors are used. Ceramic capacitors are capacitors that have ceramic materials as the dielectric and metals as conductors; electrolytic capacitors are polarized where a metal and its insulating oxide form the positive plate and the dielectric, while an electrolyte serves as the negative plate. Electrolytic capacitors have relatively large capacitance because of their thin oxide dielectric.[2]

II. Theory

Exponential Discharge

During capacitor discharge, the current flowing through a resistor depends on the voltage over the capacitor. Using Ohm's law, we can derive the differential equation[3]:

$$\frac{dQ}{dt} = -\frac{Q}{RC} \tag{1}$$

When the capacitor has an initial charge of Q_0, the differential equation has solution:

$$Q = Q_0 e^{\frac{-t}{RC}} \tag{2}$$

The solution demonstrates an exponential discharge with time constant $1/RC$ from the capacitor. By measuring voltage against time, the capacitance can be measured as $Q = CV$. By manipulating equation (2) we can get:

$$lnV = -\frac{1}{RC}t + lnV_0 \tag{3}$$

where V_0 is the initial voltage. The gradient of the line is $-1/RC$.

Low-pass Filter

The circuit used in the experiments act as a low-pass RC filter. The voltage across the capacitor has a frequency dependence, and the circuits can be seen as an impedance-divider. The frequency dependence of the input-output relationship is given by the equation[4]:

$$\tilde{V}_{out} = \tilde{V}_{in} \times \frac{\tilde{Z}_C}{\tilde{Z}_R + \tilde{Z}_C} \tag{4}$$

At low frequencies($\omega CR \ll 1$), the capacitor amplitude, given by $A = V_0/\sqrt{1+(\omega CR)^2}$, will be approximately V_0, and at high frequencies ($\omega CR \gg 1$), the amplitude will approach zero.

Capacitive Reactance

The capacitive reactance is defined as the opposition to the voltage change across a component, and it is defined as[3][4]:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} \tag{5}$$

The reactance can also be expressed by relating current to the potential drop[3]:

$$X_C = \frac{V_x}{I_g cos\phi} \tag{6}$$

where I_g and V_x are the amplitudes of the current supplied by the signal generator and the voltage across the capacitor. the phase angle ϕ is introduced as the circuit contains both resistance and capacitive reactance. The phase angle is given by[3][5]:

$$\phi = cos^{-1}(\frac{V_g sin\alpha}{V_{R1}}) \tag{7}$$

where α is the phase difference between the generator and the capacitor signal and V_{R1} is the potential difference across resistor R1. Lastly, V_{R1} is given by[3]:

$$V_{R1} = \sqrt{(V_g cos\alpha - V_x)^2 + (V_g sin\alpha)^2} \tag{8}$$

By measuring, V_x. V_g and α, the total capacitance of the circuit can be calculated:

$$C_{total} = \frac{V_g sin\alpha}{2\pi\nu R_1 V_X \sqrt{(V_g cos\alpha - V_x)^2 + (V_g sin\alpha)^2}} \tag{9}$$

III. Method

Experiment A

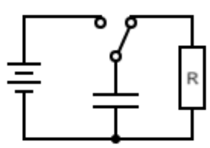
The first experiment measures the capacitance of the $1mF$ electrolytic capacitor. Since they are polarised they need to be connected in the correct orientation. A circuit is constructed according to figure 1 using a resistor with a resistance of $10.0 \pm 0.5k\Omega$[3]. An oscilloscope is set up to measure the voltage across the capacitor and a manual switch is used to charge and discharge the capacitor. The complete voltage trace of one discharge will then saved as a .CSV file.

Experiment B

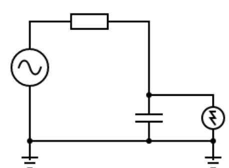
This experiment measures the $470pF$ ceramic capacitor capacitance which is smaller than that of the electrolytic capacitor. A signal generator is used for this experiment rather than physically switching between charging and discharging. The circuit will be set up according to figure 2 and an $100 \pm 5k\Omega$[3] resistor will be used. For the signal generator, a square wave with a peak-to-peak voltage of $2V$ will be used and the frequency will be adjusted until full charging and discharging can be seen in each cycle.

Experiment C

In this experiment, the goal is still to measure the capacitance of the small capacitor but a different circuit configuration, shown in figure 3, is used since the scope is now being considered as a $1M\Omega$ resistor and a $20pC$capacitor in parallel to the capacitor[3]. The circuit used for this experiment will be the same as the circuit used for experiment B but replaced with a $6.8 \pm 3k\Omega$ resistor. A $2V$ peak-to-peak sine wave is generated at a frequency of $50kHz$[3]. The mean values of V_x, V_g, and α and their associated uncertainties are calculated using the oscilloscope statistics, and the voltage trace is stored for analysis.



(a)



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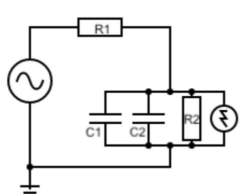


(c)

Fig. 1: The circuit setups for (a) experiment A, (b) experiment B, (c) experiment C

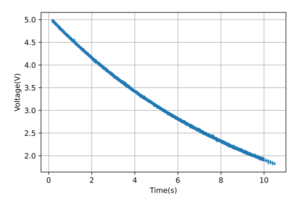
IV. Results

Experiment A

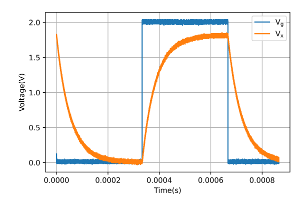
Figure 2(a) shows the change in voltage across the capacitor during one discharge and a clear exponential decay can be observed. At $t = 0$, the voltage across the capacitor is approximately $5V$. As time increases, the rate of decrease of voltage across the capacitor started to decrease. The voltage dropped by 3 in around 10 seconds. Figure 3(a) shows the change in the natural log of voltage, $ln(V)$, against time, t, and the line is straight with a negative gradient. At $t = 0$, the value of $ln(V)$ is around 1.6. The Capacitance of the electrolytic capacitor, calculated from the gradient of the line of best fit is $1.03 \pm 0.01mF$. The calculated result is 3% larger than the stated value of the capacitance.

Experiment B

For this experiment, the discharging part on figure 2(b) will be analyzed. The curve also shows an exponential decay similar to experiment A. However, the time span is much shorter as the discharge decay is around 0.0003 seconds. In figure 3(b), the plot of the logarithm voltage trace is narrow from 0 to 0.0001 second and then started to spread out as time increased further. This is caused by the fluctuation of the discharge voltage near 0V. The capacitance of the ceramic capacitor, calculated from the gradient of the line of best fit after error propagation, is $539 \pm 10pF$. The calculated result is 15% larger than the stated value of the capacitance.



(a)



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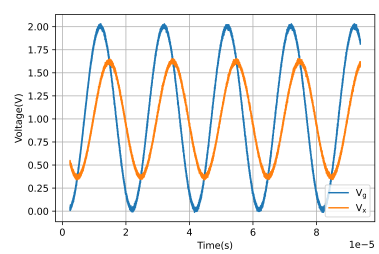
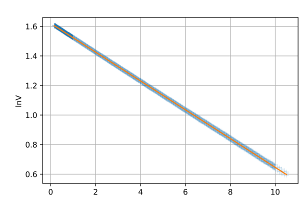
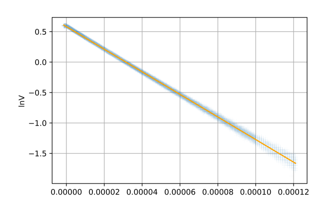


(c)

Fig. 2: Voltage traces recorded on oscilloscope for (a) experiment A, (b) experiment B, (c) experiment C, plotted with python



(a) Experiment A



(b) Experiment B

Fig. 3: Plots of $ln(v)$ against t for one cycle of capacitor discharge with lines of best fit, plotted with python

Experiment C

In figure 2(c), the orange wave has a lower amplitude and has a phase difference α to the blue wave. The mean values of V_g and V_x are $2.052 \pm 0.006V$ and $1.319 \pm 0.005V$ respectively. The mean phase difference is 50.6 ± 0.6^o. By using equation (9), the total capacitance in the circuit can be calculated. By postulating that the oscilloscope has a capacitance of $20pF$, the capacitance of the ceramic capacitor, after error propagation, is $330 \pm 20pF$.The calculated result is 30% smaller than the stated value of the capacitance.

Analysis

The capacitance calculated from both methods have similar values as they are on the same order of magnitude. Both methods produced similar uncertainties: $\pm 20pF$ for the phase shift method and $\pm 10pF$ for the discharge decay method. The phase shift method is more appropriate for determining the capacitance of the small capacitor because the oscilloscope's capacitance is also considered in the calculation, which is important when measuring a small capacitance. For comparison, the value stated on the capacitor is $470pF$ which is within the same order of magnitude of the measurements.

Further Discussion

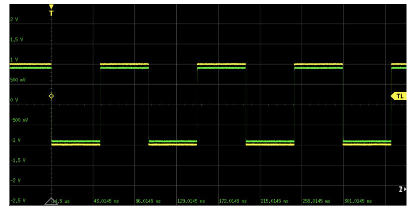
The circuit used for experiment B can be seen as a low-pass RC filter, where the amplitude of the capacitor potential difference depends on the input voltage, frequency, capacitance, and resistance. By rearranging the formula, the capacitance can be expressed as:

$$C = \sqrt{\frac{V_0^2 - A^2}{(2\pi\nu A)^2}} \tag{10}$$

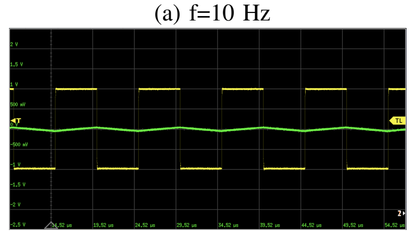
When the frequency is $1500Hz$, the peak-to-peak voltage of the signal and the capacitor are 2.0727 and 1.8865V respectively. The capacitance calculated from equation 10 is $440 \pm 20pF$. The calculated result is 6% smaller than the stated value of the capacitance. The theory and the results from both the voltage trace and the calculated capacitance suggests that this is an appropriate model to calculate the capacitance.

V. Conclusion

This investigation aims to investigate the capacitance of capacitors using different methods. For the discharge decay method the oscilloscope voltage traces are exported as .CSV files and converted into logarithm graphs using python, where the gradients are proportional to the capacitance; for the phase shift method, the phase difference, peak-to-peak voltages of the signal and the capacitor and their uncertainties are obtained through the statistics function on the oscilloscope. The calculated capacitance for the electrolytic capacitor is $1.03 \pm 0.01mF$. The ceramic capacitor capacitance obtained through the discharge decay and phase shift methods are $539 \pm 10pF$ and $330 \pm 20pF$ respectively. Lastly, the low-pass filtering nature of the RC circuit is discussed and an alternative method to calculate the capacitance is proposed, giving a value of $440 \pm 20pF$, which is the closest to the stated value on the capacitor.



(a) f=10 Hz



(b) f=100 kHz

Fig. 4: Oscilloscope voltage traces of the generator and capacitor at different input frequencies. The yellow wave is the signal generator voltage trace and the green wave is the capacitor voltage trace

References

[1] Tipler, P. and Mosca, G., 2003. Physics for scientists and engineers. 6th ed. New York: W.H. Freeman, pp.802-803.

[2] Tipler, P. and Mosca, G., 2003. Physics for scientists and engineers. 6th ed. New York: W.H. Freeman, pp.810-812.

[3] S. P. D. Mangles (Ed.) Year 1 Laboratory Manual: Measuring Capacitance', Imperial College London, 2021

[4] Tipler, P. and Mosca, G., 2003. Physics for scientists and engineers. 6th ed. New York: W.H. Freeman, pp.1002-1003.

[5] Powell, R., 1995. Introduction to electric circuits. London: Arnold.

Appendix

The python code for fitting the best fit lines form the oscilloscope data:

```
t_A, v_A = np.loadtxt("ExA.csv",delimiter=
',',skiprows=9000,unpack=True)
lnv=[]
for i in v_A:
    x=np.log(i)
    lnv.append(x)
def line (t,v_0,C):
    return np.log(v_0)-t/(10000*C)
g_v_0=1.6
g_C=10**-3
p0=[g_v_0,g_C]
para, cov = sp.optimize.curve_fit(line, t_A
, lnv, p0, absolute_sigma=True)
t_B, v1, v2=np.loadtxt("ExBnewlab.csv",deli
miter=',',skiprows=8000,unpack=True)
lnvb=[]
for i in v2:
    x=np.log(abs(i))
    lnvb.append(x)
para, cov = np. polyfit(t_B,lnvb,1,cov=True)
```