Measuring the Maximum Beta Decay Energy of Sr-90 with the Energy Loss Method

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***Abstract*—The maximum beta decay energy of Sr-90 was calculated using the energy loss method with copper and aluminum sheets. Results show maximum energies of $1.921 \pm 0.212MeV$ and $2.062 \pm 0.230MeV$ for copper and aluminum. The method and uncertainty analysis were shown to be effective, however, longer sample times should be used for future experiments to increase accuracy.**

I. INTRODUCTION

The study of beta decays is pivotal for learning the behavior of atomic nuclei. This paper presents an approach to measure the maximum beta decay energy for Sr-90, with a half life of 28.8[1] years. The method utilises the energy attenuation of beta particles in copper and aluminium. The experimental setup and methodology applied for obtaining precise measurements of the maximum energy are included. The results and their implications are then discussed.

II. METHOD

A. Experimental Setup

Fig.1 shows the experimental setup consisting of a sealed Sr-90 source and a silicon detector, clamped to a vertical rod. The source and the detector are $29.020 \pm 0.05mm$ away from each other. There is a detecting window on the silicon detector and the metal sheets are stacked on top of the window such that it is completely covered. The silicon detector is connected to a computer with a cable which transfers the signals to the counter program.

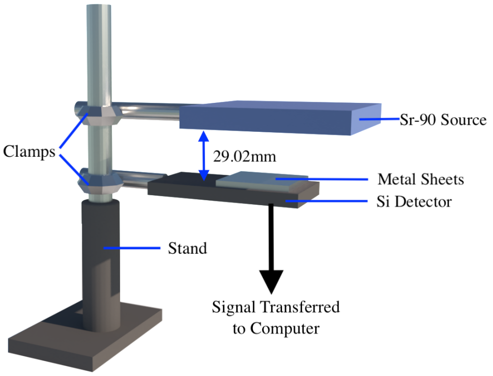

Fig. 1. Experimental setup, the silicon detector is connected to a computer. The thickness of the metal sheets vary but the separation between the source and detector always remains constant.

B. Data Collection

There are 16 metal sheets for both copper and aluminium of various thicknesses. Their thicknesses are measured individually with a micrometer screw gauge and labeled with numbers 1 to 16. The corresponding thickness for a specific combination of sheets was calculated by adding their values together and the total uncertainty was calculated by error propagation of addition. The sample time was chosen be 30 seconds. A rather long sample time was used to reduce the standard deviation for the count measurement obtained, which was caused by the random nature of radioactive decay[2].

C. Fitting the data

The data were fitted using curve fit from the Scipy optimization library. The function that the data would be fitted to was defined as piece-wise linear so the final fit would have three segments only. The parameter uncertainty was calculated using the covariance matrix and the fitting uncertainty was approximated by first randomly selecting data points from the original data and fitting the lines repeatedly (n=500) to find the mean range. The reasons for choosing a piece-wise linear function are discussed in the next section.

III. THEORY

A. Finding the Best Fit

The critical thickness was found by finding the second turning point of the best fit piece-wise linear function. The energy of the beta particles emitted ranges theoretically from 0 to $2.27MeV$, however the detector cannot measure energies that are small due to its efficiency. Hence, the relationship between the stopping power and the energy of beta particles is approximated as linear[3]. For photons, which are also emitted by Sr-90, the relationship between log count rate and thickness is linear because the Sr-90 photons are mono-energetic[3]. Therefore, the plot of log count rate against thickness must be fitted with piece-wise linear functions.

B. Systematic Uncertainty

The count rate has five main areas of systematic uncertainty: dead time of the detector, background radiation, energy attenuation of beta particle in air, and detector efficiency. The dead time uses the nonparalyzable model[2] for signal counting and the overall fraction of count lost is $1 - e^{-mt_D}$, where m is the mean count rate and t_D is the dead time. The background radiation is obtained from a $300s$ measurement, resulting in $0.123 \pm 0.002Bq$. For energy attenuation in air, since the distance between the source and detector is fixed, the thickness of air decreases as the number of metal sheets increases. The Bethe formula gives:

$$\frac{d\bar{E}}{dx} = 0.306\rho\frac{Z}{A}\beta^{-2}ln(\frac{1.16E}{I}) \tag{1}$$

For air, the density, ρ, is $1.293 \times 10^{-3} gcm^{-3}$[4], and the Z and A values calculated by taking a weighted average of elements in air[5] gives 7.8 and 14.4, respectively. The rate of energy attenuation (dE/dx) is approximately proportional to $\rho\frac{Z}{A}$[3]. There was a 0.05% and 0.02% change in the rate of energy loss for aluminium and copper after considering energy attenuation in air. Lastly, the detector efficiency also introduces a systematic error as beta particles with energies below a threshold could not be detected. However, this quantity cannot be estimated with the data obtained from the experiments.

IV. RESULTS AND ANALYSIS

Figure 2 shows the log count rate against thickness data, piece-wise connected best fit function and uncertainties for copper and aluminium. As expected, the best fit function for both copper and aluminium exhibit three distinct gradients. Corresponding to the systematic uncertainty analysis for the count rate, the data points for copper and aluminium have similar uncertainties, which increases as the logarithm of the count rate decreases. The thickness uncertainties of the data points are comparable between the two plots, although they appear larger for the copper plot due to the difference in x-axis scale. Furthermore, the orange parameter uncertainty, calculated from the covariance matrix of the best fit function, increases as the thickness of the material increases. This indicates that the data at larger distances from the source have larger fluctuations. Notably, the blue total uncertainty for both plots tend to be large in the beginning and the end of the x-axis. The overall uncertainty is also particularly large at the second turning point, meaning that the fitting in this region has a large variability. However, the overall shapes of the uncertainties for both plots closely follow the shape of the best fit lines, demonstrating a high degree of fitting accuracy. The critical thickness is $1.00 \pm 0.01mm$ for copper and $3.59 \pm 0.04mm$ for aluminium. The maximum energy of the beta spectrum can be determined using:

$$R = 0.11(\sqrt{1 + 22.4E_{max}^2} - 1) \tag{2}$$

Where R is the mean range of the electrons with unit gcm^{-2}, which is defined as the track length of a particle that constantly loses energy at the mean rate[3]. Figure 3 shows the log count rate for both metals against the areal density. The best fit function for copper is always below the that for aluminium, which means that the stopping power of copper is higher than the stopping power of aluminium for both electrons and photons. In the first two segments of the best fit function, the gradient of copper is steeper than the aluminium gradient, meaning that more electrons are absorbed as the areal density increases. For the last segment, most of he counts are from photons. The copper line is lower than the aluminium line, which means that more photons are absorbed in copper. Energy loss is a probabilistic event, there will be a spread about the mean range called straggling, which is around 10 percent of the mean range[3]. The mean range for copper and aluminium after accounting for straggling are $0.896 \pm 0.099gcm^{-2}$ and $0.969 \pm 0.108gcm^{-2}$ respectively. The maximum energy for the beta spectrum calculated using Eqn.2 for copper and aluminium are $1.921 \pm 0.212MeV$ and $2.062 \pm 0.230MeV$ respectively. They are both smaller than the theoretical value of $2.27MeV$. One possible reason for measuring a lower energy than expected is the relatively short sample time of 30 seconds used in the experiment. This decreases the likelihood of observing beta decays with close to maximum energy, which make up only a small fraction of the total energy spectrum.

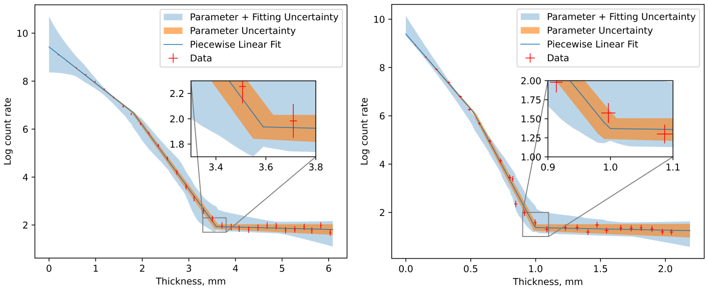

Fig. 2. Log count rate against thickness of aluminium(left) and copper(right) with parameter and total uncertainties. The systematic errors have been taken into account. The zoom-in shows the turning point of the fitted piece-wise function.

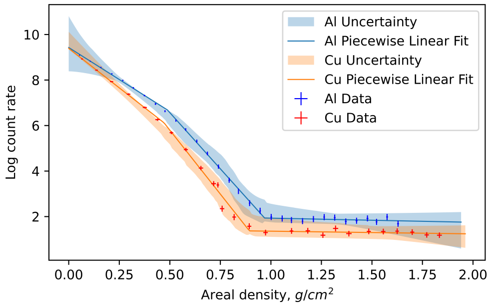

Fig. 3. Log count rate against areal density for aluminium and copper with their total uncertainties. The x uncertainty for copper is larger because it is multiplied by the density of copper, which is higher than that of aluminium

V. CONCLUSION

The Maximum energy for the beta spectrum of Sr-90 calculated using the energy loss method for copper and aluminium are $1.921 \pm 0.212MeV$ and $2.062 \pm 0.230MeV$ respectively. They are mostly agree with the theoretical value (15.4 and 9.2 percent difference respectively). Rigorous uncertainty analysis for both statistical and systematic uncertainty increased the efficiency of this technique. A longer sample time should be adapted in future experiments to increase the accuracy of the mean range.

REFERENCES

[1] "Standardization of 90Sr (SRM 4329a)," NIST, Aug. 2022, Available: https://www.nist.gov/pml/radiation-physics/standardization-90sr-srm-4329a

[2] Glenn Frederick Knoll, Radiation detection and measurement. New York, Ny: Wiley, 2000.

[3] N. J. Carron, An Introduction to the Passage of Energetic Particles through Matter. CRC Press, 2006.

[4] N. Earth Science Data Systems, "Air Mass/density," Earthdata. https://www.earthdata.nasa.gov/topics/atmosphere/atmospheric-pressure/air-mass-density: :text=Pure

[5] C. W. Allen and A. N. Cox, Allen's astrophysical quantities. New York: Aip Press, 2000, p. 258.